The speed of drifting bodies in a stream

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SUMMARY

The speed of free floating bodies on the surface of a water stream in a sloping channel has been found to be sensibly the same as the mean speed of the layer of water in which the bodies are floating, contrary to some recorded opinions.

It is sometimes believed that a ship drifting in a river without power or sail travels faster than the mean speed of the layer of water in which it is floating; it is said that the relative speed of the water past the rudder enables the ship to be steered. Prandtl (1952, p. 179) for example says " The boat, in fact, hurries ahead of the water, and gets enough way on her to be steered ".

The basis of the belief is that the weight force W of the ship is directed vertically downwards, but that the hydrostatic buoyancy force on the ship is directed at right angles to planes of equal pressure, that is, normal to the free surface. Since the surface in a river in uniform flow is inclined by friction at an angle i to the horizontal, it follows that a component force Wi ($i \doteqdot \sin i$ for usual slopes) acts on the ship in a direction down the surface slope. This component force is balanced in steady motion by the hydrodynamic drag force due to the relative motion between ship and water. Alternatively, it is believed that a ship travels faster than the equivalent volume of water because there is an exchange of momentum by turbulent motions across the boundaries of the water. Since the mean speed of the stream decreases with depth, the exchange of momentum causes a resistive force to be exerted on the water volume. The solid boundary of the ship prevents this exchange, so that the only forces on the ship restraining the down-gradient motion are shear stresses caused by the motion of the ship relative to its surroundings, it being tacitly assumed that the flow and pressures in the surroundings are not affected by the replacement of the water volume by the ship. If, in fact, the solid boundary modifies the flow in the neighbourhood, then the forces on it may be quite unlike those at the boundaries of the same volume of water. The buoyancy force on the ship may not then be normal to the free surface, so that the down-hill force can no longer be so easily found.

It is difficult to show if ships do indeed travel faster than their surroundings; for the effect must be small in slow flowing rivers (with small i), and it is clearly imprudent to allow a ship to drift unrestrainedly in a fast and turbulent river (which has a larger i). It is more practical to test the existence of this relative velocity on a cylinder floating with its axis vertical in a stream. Such floats are sometimes used by engineers to determine the speed of a river, and it may be desirable to ascertain if there is any systematic error in using them for this purpose.

The force W on such a cylinder of diameter d and length l when floating in a fluid of density ρ is $W = \frac{1}{4}\pi d^2 l \rho g$. If the cylinder is moving at a velocity u_{re} relative to its surroundings, then the drag force F on it is approximately that which the same cylinder would experience if it was travelling at the same speed relative to still water. On this approximation, $F = C_d dl(\frac{1}{2}\rho u_{rel}^2)$; and, equating F to the downstream component of W, we have

$$C_d dl(\tfrac{1}{2}\rho u_{rel}^2) = \tfrac{1}{4}\pi d^2 l \rho g i,$$

where C_d is the coefficient of drag of the cylinder. It is convenient to substitute the frictional properties of the river channel for the variable i, and one empirical formula used by engineers is

$$\bar{u} = C\sqrt{(mi)},$$

where C is a coefficient (Chezy's coefficient) predominantly determined by the roughness of the channel, $\bar{u}$ is the mean velocity, and m is the hydraulic mean depth, i.e. the cross-sectional area divided by the wetted perimeter of the channel (equal to the depth if the channel is wide). Substituting for i in the previous equation, we obtain

$$\frac{u_{rel}}{\bar{u}} = \frac{1}{C}\sqrt{\left(\frac{\pi g d}{2mC_d}\right)}.$$

It will be seen from this equation that a high value of u_{rel} should be obtained in rough-walled channels (having a low value of C) and with cylinders having a large d/m ratio. Engineer's floats may be cylinders of about 3 in. diameter, and might well be used in a typical river of depth $m = 10$ ft., $C = 100$ ft.$^{1/2}$ sec^{-1}. The equation shows that the theoretical value of $u_{rel}/\bar{u}$ is about 0·01; it is probable that this small increase of velocity (if it exists at all) is hidden by experimental error and by the large-scale turbulence of the stream.

The point may be better investigated in a laboratory channel which can be made to have a great roughness, and experiments have been so made in the Civil Engineering Laboratories of Imperial College. Systematic roughness elements, of a type proposed by Denil and tested by White & Nemenyi (1942), were arranged on the bottom of a glass sided channel 11 m long and 30 cm wide. These roughnesses are zigzag walls 1·6 cm high running across the channel. From alternate 90° angles, longitudinal walls of the same height connect one wall with the next, 7·5 cm upstream or downstream. The bed of the channel may be tilted so that it is parallel to the surface of the water. Four floats were made, all circular cylinders ballasted to float to a draft of 2·5 cm with their axes vertical. A quantity of 0·00652 m^3 sec^{-1} was made to flow down the channel; and it was found that if the bed was inclined at $i = 1/271$, the depth was constant at 7·5 cm above the top of the roughness. Under these conditions, $C = 21$ m$^{1/2}$ sec^{-1} for

$\bar{u} = 0{\cdot}286$ m/sec. The theoretical value of $u_{rel}/\bar{u}$ for each cylinder is given in table 1. In this table the coefficients of drag have been taken as those applicable to cylinders whose length-diameter ratio is double that of the cylinders tested, so as to allow for the three-dimensional flow round the lower end (Goldstein 1938, p. 439).

Float diameter (cm) :	6·3	1·9	0·7	0·2
Calculations				
Length/diameter	0·4	1·3	3·6	12·5
Drag coefficient	0·6	0·7	0·8	0·95
$u_{rel}/\bar{u}$	0·270	0·137	0·078	0·038
u_{rel} (cm/sec)	7·7	3·9	2·2	1·1
Observations				
Number of successful drifts	20	21	22	16
Mean time to traverse 2·88 m (sec)	6·83	6·90	6·80	6·89
Mean deviation (sec)	±0·24	±0·31	±0·25	±0·24
Mean speed (m/sec)	0·421	0·417	0·423	0·418

Table 1. Calculated and measured velocities of cylinders 2·5 cm long drifting in a stream of mean velocity 0·286 m/sec.

The observed speeds of the floats are also shown in table 1. They were timed to the nearest 1/10 sec with a stopwatch calibrated in 1/100 sec, over a 2·88 m length of the channel, starting 5 m downstream of the channel inlet and finishing 3 m upstream of the outlet. The floats were released 1 m upstream of the starting line in mid-stream; and, if the float later drifted so that its axis came outside of the middle third of the channel, its time was disregarded. Only about 1 float in 6 drifted successfully within the middle third.

It will be seen that the calculation predicts that the largest cylinder should travel some 6·6 cm/sec faster than the smallest, but that the observations do not disclose any significant difference in speed for the large range of d/m used.

It is, however, possible that *all* the cylinders drifted at the same speed relative to the upper 2·5 cm of water, in which they all were immersed. Two tests were made to explore this possibility. In the first, drops of dye were put into the stream just ahead and around the largest cylinder. On no occasion did the cylinder appear to overtake the dye and to float into clear water. If the relative velocity exists, then it should have overtaken at 7·7 cm/sec, a speed easily observed.

In the second test, carried out concurrently with the timing of the floats over the 2·88 m test distance, the mean speed of the upper 2·5 cm of water was measured by a current meter. This was a paddlewheel, 30 cm radius, rotating on a horizontal axis, with light aluminium blades immersed to a depth of 2·5 cm. The part of the wheel in the air was protected from draughts. The meter was calibrated before and after the test by towing it through still water. The mean speed of the water was found five times during the timing of the floats both upstream and downstream of the test length. The mean water speed so obtained was 0·420 m/sec; this was close to the mean speed of all the floats (0·420 m/sec $\pm$ 0·017 m/sec mean deviation of 79 observations).

It therefore appears that the floats were in fact travelling, as nearly as could be measured, at the same speed as the water; and that the 'hurrying ahead' of the float was either not present at all, or was much smaller than predicted. Perhaps the presence of shear in a stream rearranges the hydrostatic pressure on the float, so that the resultant upthrust exactly balances the weight force; in this respect, a drifting body therefore appears to affect the flow in the surrounding fluid. This is fortunate, for many experiments in fluid mechanics are carried out using particles of near neutral buoyancy as tracers. If the gradient effect had been real, then systematic errors would occur if pressure gradients occurred. In meteorology, pilot balloons would not travel with the wind; since the atmospheric pressure gradient is at right angles to the wind, a sideways component would be given to the balloon, giving a false direction. The above experiments should not be taken to apply to a flow which has a surface gradient but no shear, in which case a floating object might possibly move relative to the water down the gradient.

If, as was mentioned at the beginning, ships can indeed be steered while drifting down a river, then a possible explanation independent of the 'hurrying ahead' effect may be that the ship's draft is nearly the same as the depth of the river. The lower part of the rudder is therefore in water which is considerably retarded by the boundary layer of the river bed. The ship travels at the mean speed of the upper layers, which is faster, so that there is relative motion over the rudder and the ship is steered. The effect will be enhanced if the ship is of small draft, with a rudder projecting well below its bottom. Tests carried out in the laboratory channel show in fact that a model boat can be steered when drifting if it has such a deep rudder. It cannot be steered if the same rudder is arranged so that it does not project below the boat.

I am indebted to Professor C. M. White, who pointed out that Prandtl's statement was untested.

References

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